Technical Comments

Comments on the Two Notes "Correct Formulation of Airfoil Problems in Magnetoaerodynamics"

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IN a recent paper,[1] Dragos has returned to the question we clarified[2] some time ago. In reference to our paper he states that "... the kernels of the integral equations to which the solution of the problem is reduced are expressed by divergent integrals (e.g., $I\pm$). Under such conditions the solution given by the mentioned authors cannot be valid."

The first part of this statement is correct; the second is not. Such divergent integrals are generalized functions (distributions), as he himself recognizes in discussing the alternative formulation which we gave. Both of our solutions, if this term can be used for the reduction of a problem to an integral equation, are valid. Since the purpose of his Note could be misunderstood, it may be useful to explain exactly what it contains.

He starts by rederiving the integral equation for the pressure jump which we gave in our paper. The method is the same; the notation is slightly changed (his kernel K is our $-2J$). He then notes that the kernel is given by a divergent integral and interprets it as a generalized function. For supersonic motion the resulting equation is of Fredholm type, whereas for subsonic motion he uses a standard technique[3] to make it so.

In short, our paper stands. But Dragos has made the very interesting observation that there is always a Fredholm integral equation underlying our treatment of the crossed-fields case.

We take the opportunity of correcting a few typographical errors in the two Notes.

***Our note*[1]**

In b below Eq. (4): $A^2 - \beta$ should read $A^2 - \beta^2$. In $H\pm$ below Eq. (15): q should read λ. In $I\pm$ below Eq. (15): $iRm\lambda^{-1}$ should read $(iRm\lambda)^{-1}$. In J below Eq. (16): $(\lambda^{-2}A^2 - 1)$ should read $(\lambda^{-2}r^2 - 1)$ in the first bracket, and $(\lambda^{-2}s^2 - 1)$ in the second.

***His note*[2]**

In Eq. (24):

$$\int_{-\infty}^{+1} \text{ should read } \int_{-\infty}^{\infty}$$

In Eq. (31): F should read f. In Eq. (31′): n should read m.

References

[1] Dragos, I., "Correct Formulation of Airfoil Problems in Magnetoaerodynamics," *AIAA Journal*, Vol. 7, No. 10, Oct. 1969, pp. 2014–2016.

[2] Fan, D. N. and Ludford, G. S. S., "Correct Formulation of Airfoil Problems in Magnetoaerodynamics," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 167–169.

[3] Muskhelishvili, N. I., *Singular Integral Equations*, P. Noordhoff, Groningen-Holland, 1953, pp. 335–336.

Received February 13, 1970. This research was supported partly by the U. S. Army Research Office, Durham and partly by the National Science Foundation under Grant GP-8711.

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Reply by Author to D. N. Fan and G. S. S. Ludford

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IN their comments on my paper, D. N. Fan and G. S. S. Ludford refer to the formal aspect of the problem.

1) The statement I made in Ref. 1, according to which the solution given by Fan and Ludford[2] cannot be valid, was based upon the observation that the general solution given by the authors:

$$u(x,y) = (2\pi)^{-1/2} \int_{-\infty}^{+\infty} \bar{u} \exp(-i\lambda x)d\lambda, \ldots, \quad (1)$$

[where $\bar{u}$, $\bar{v}$, $\bar{p}$ are determined with the aid of formulas (7) and (8)[2]] has to be modified. In case this modification is not effected, the functions $\bar{u}$, $\bar{v}$, $\bar{p}$ no longer remain bounded for $|\lambda| \to \infty$ (the behaviour of r^2 and s^2 is similar to that of λ^2 for a large λ) and therefore the Fourier integrals (Eq. 1) cannot be defined. Consequently, this form of the solution cannot be valid and hence the respective paper has to be entirely remade.

The authors agree with my first statement that the kernels of the integral equation (e.g., $I_\pm$ in Ref. 2) are divergent. The study of these integrals and their reduction to convergent integrals by using the theory of distributions, is however not to be found in Fan and Ludford's paper. In the absence of such a study it is difficult to make out the significance given by the authors to the results expressed by divergent integrals.

2) Fan and Ludford state that the method used for the solution of the problem is the same in both Ref. 1 and Ref 2. It is, however, evident that, in essence, the method used by these authors is in fact the one I had used previously in my papers[3] for the solution of this problem. This method is based upon the representation of the solution by Fourier integrals and upon its reduction to an integral equation.

Two important remarks have however been made by the authors in Ref. 2. The first refers to the magnitudes u, v, h_x ... which satisfy by themselves the equation I used in Ref. 3 for ω and j; this remark leads to a simplification of the solution (as a matter of fact, in Ref. 3 I had already remarked that the potential part of the solution was null). The second remark states that the integral equation given in my paper[3] is not sufficient for the solution of the problem, a supplementary condition being required (the pressure continuity on the axis Ox in the fluid). These two remarks have been taken into account in my last solution[1].

References

[1] Dragos, L., "Correct Formulation of Airfoil Problems in Magnetoaerodynamics," *AIAA Journal*, Vol. 7, No. 10, Oct. 1969, pp. 2014–2016.

[2] Fan, D. N. and Ludford G. S. S., "Correct Formulation of Airfoil Problems in Magnetoaerodynamics," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 167–169.

[3] Dragos, L., "Reply by Author to P. Greenberg," *AIAA Journal*, Vol. 5, No. 7, July 1967, pp. 1367–1370; also "L'ecoulement d'un Fluide a Conductivite Electrique Finie, en Presence d'un Profil Mince," *Atti Dei Lincei*, Vol. 42, 1967, p. 381.

Received April 3, 1970; revision received May 25, 1970.

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